Mokken scale analysis : A non parametric item response model

Jean-Luc Kop

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Evaluation et Analyse de la Qualité de Vie en Oncologie
Nouveaux Développements Méthodologiques
Cancéropôle du Grand Sud-Ouest
Montpellier, 3-4 avril 2014

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A three-item example

During the last week…

…did you feel sad?

… have you cried ?

… had suicidal thoughts?

Why do one answers “yes” or “no” to these questions ?

What is needed is a theory of item response…

an Item Response Theory (IRT)

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The beginning of a theory

…did you feel sad?

… have you cried ?

… had suicidal thoughts?

- There is a unique cause which is solely responsible for the responses to these items
- This unique cause is « psychological distress »
- Psychological distress is a latent continuum that differentiates individuals

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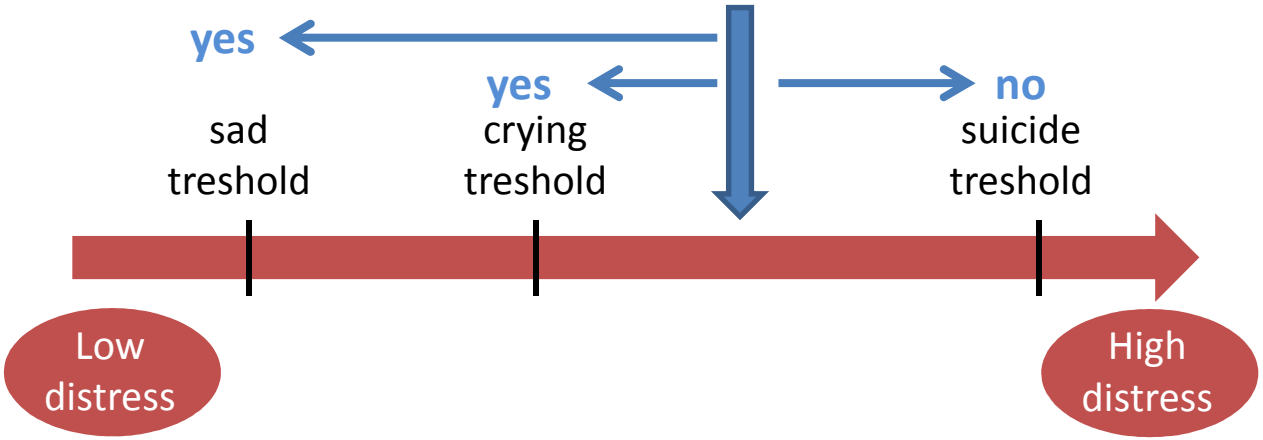
A rather strong theory

…did you feel sad?

… have you cried ?

… had suicidal thoughts?

- below a given threshold on the continuum, the answer is always negative, and above this threshold, the response is always positive



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Guttman's scalogram (a deterministic IRT)

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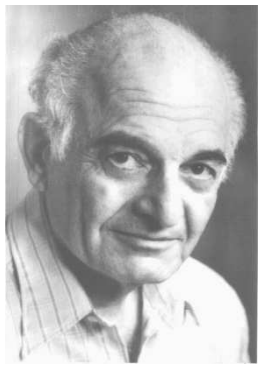

Louis Guttman
1916-1987

	sad	crying	suicide
Very low	0%	0%	0%
Low	0%	0%	0%
Medium	100%	0%	0%
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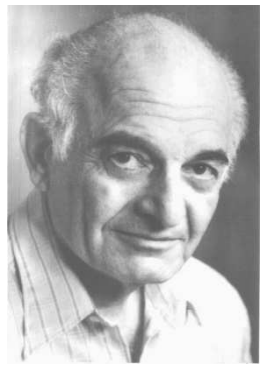

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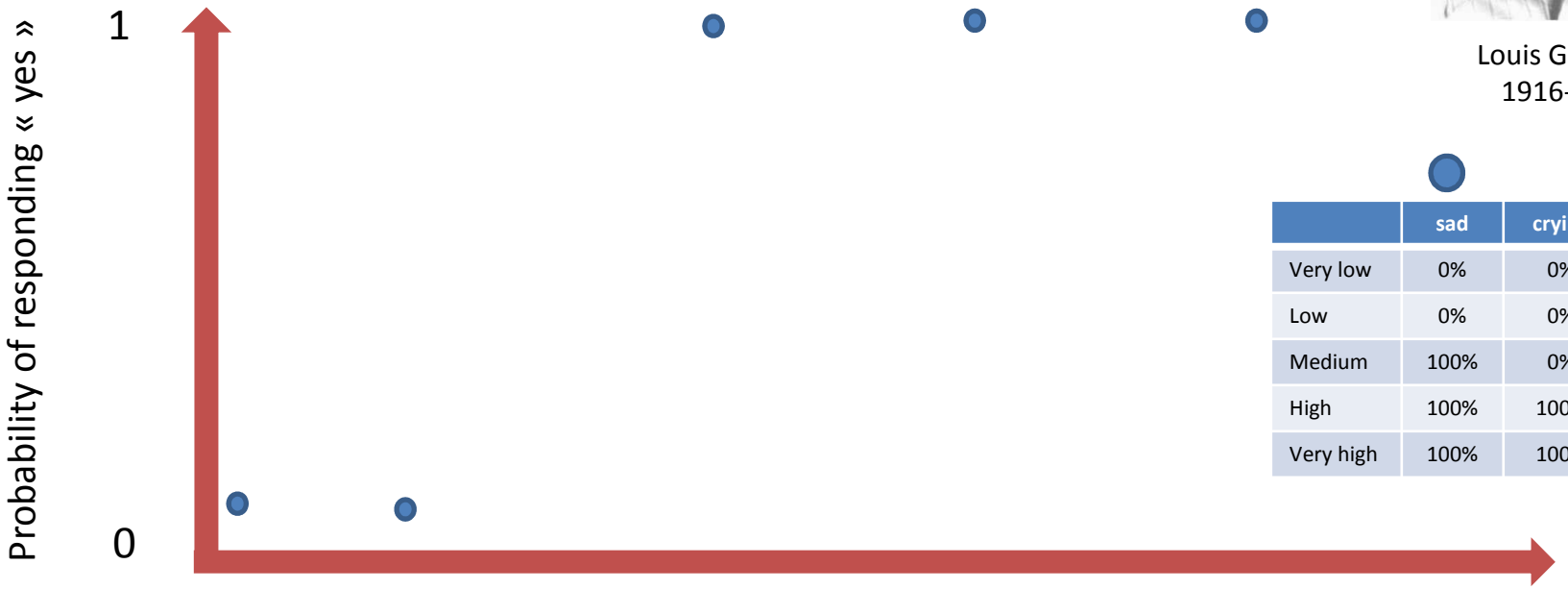

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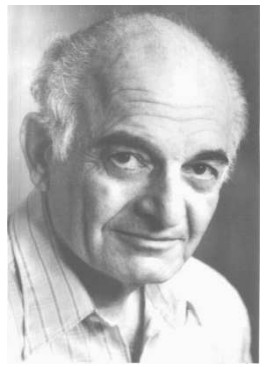

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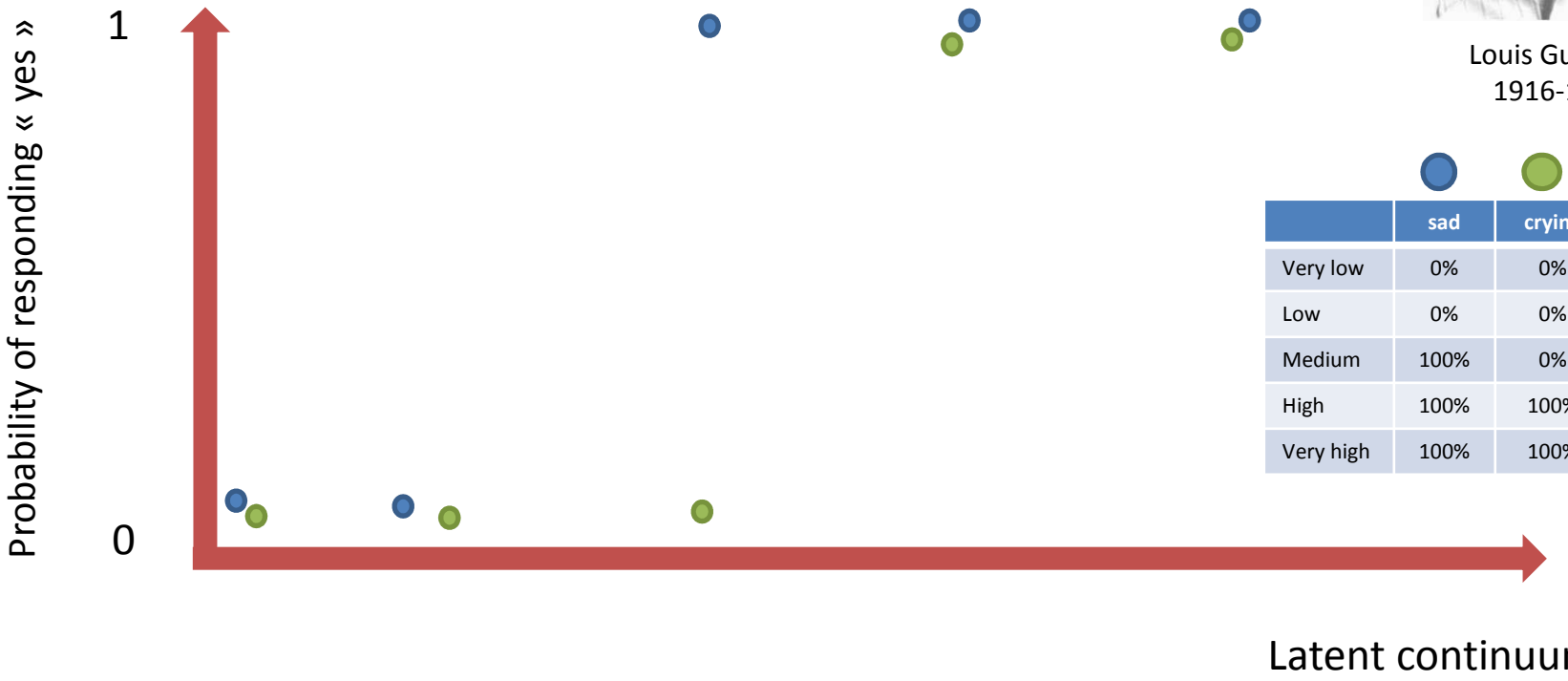

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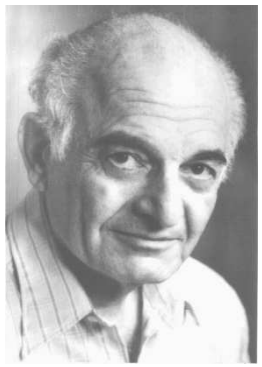

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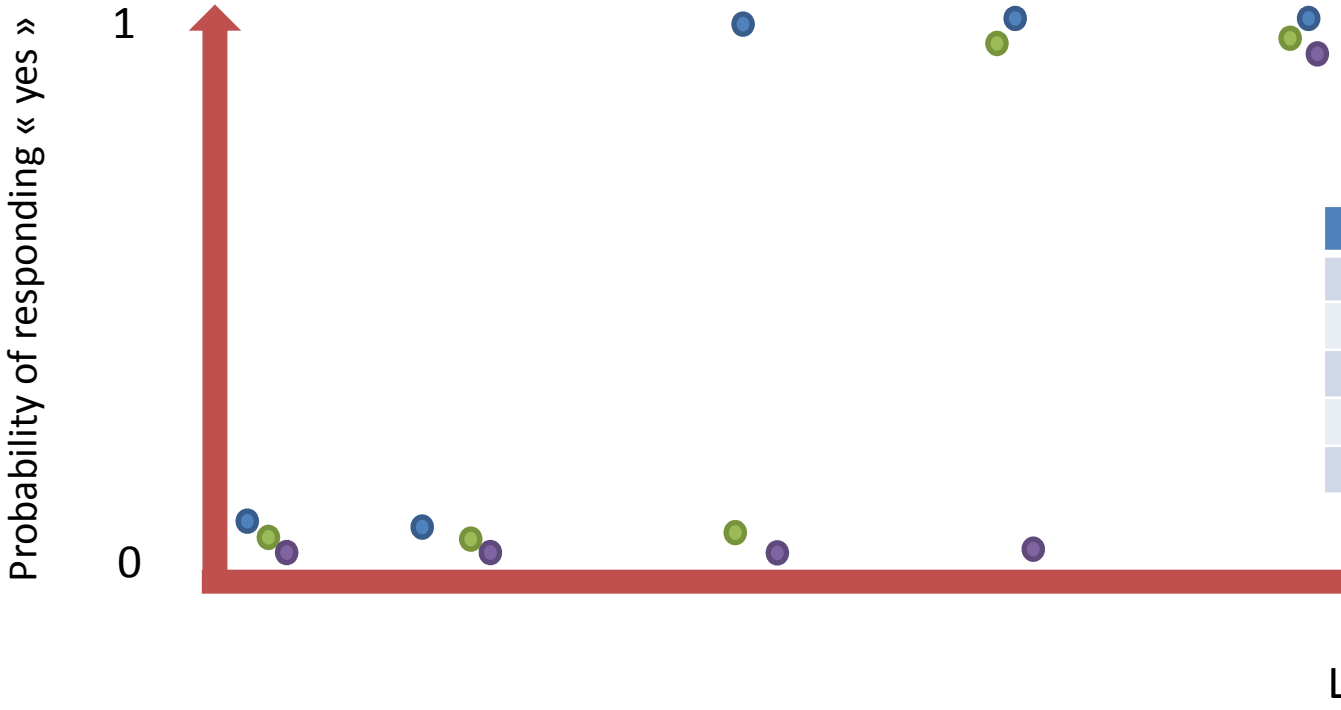

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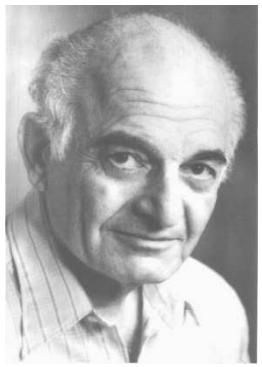

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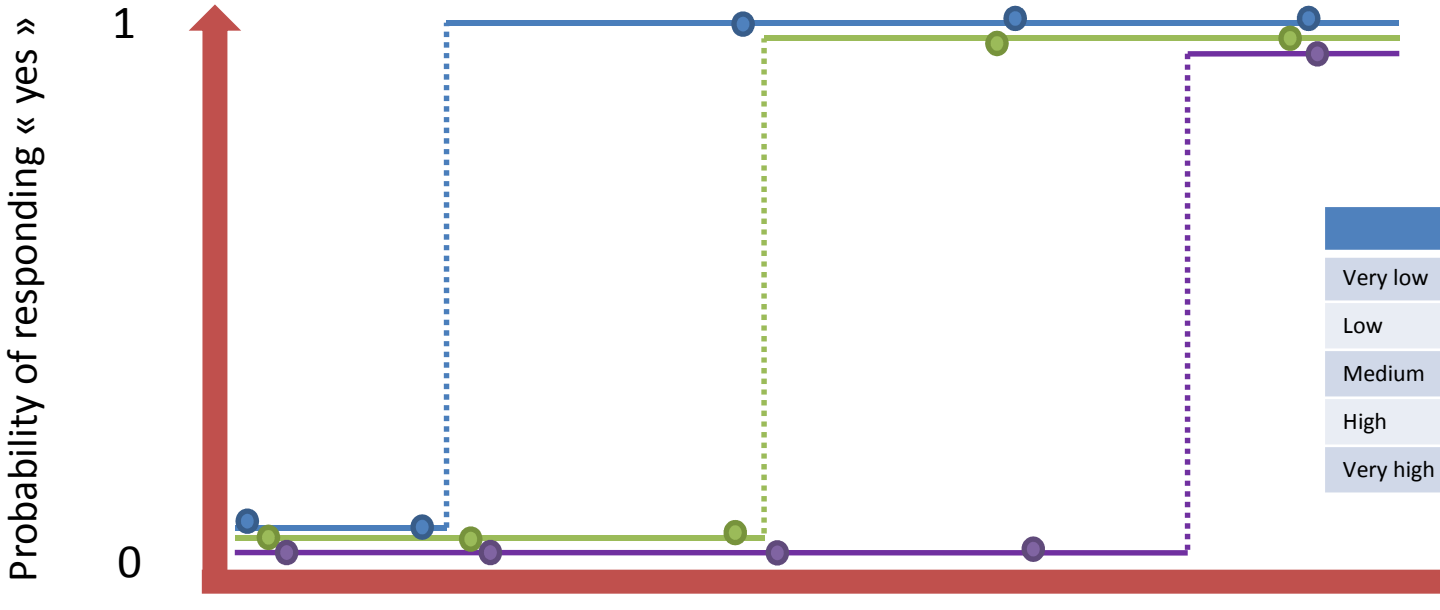

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A less restrictive theory

- The probability of a “yes” answer depends mathematically of the position on the continuum but it is not necessarily 0 or 1

	sad	crying	suicide
Very low	12%	0,7%	0,1%
Low	50%	5%	1%
Medium	88%	27%	7,6%
High	98%	73%	38%
Very high	99,8%	95%	82%

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The Rasch parametric model

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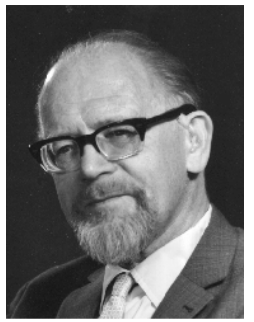
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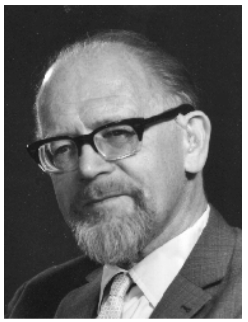
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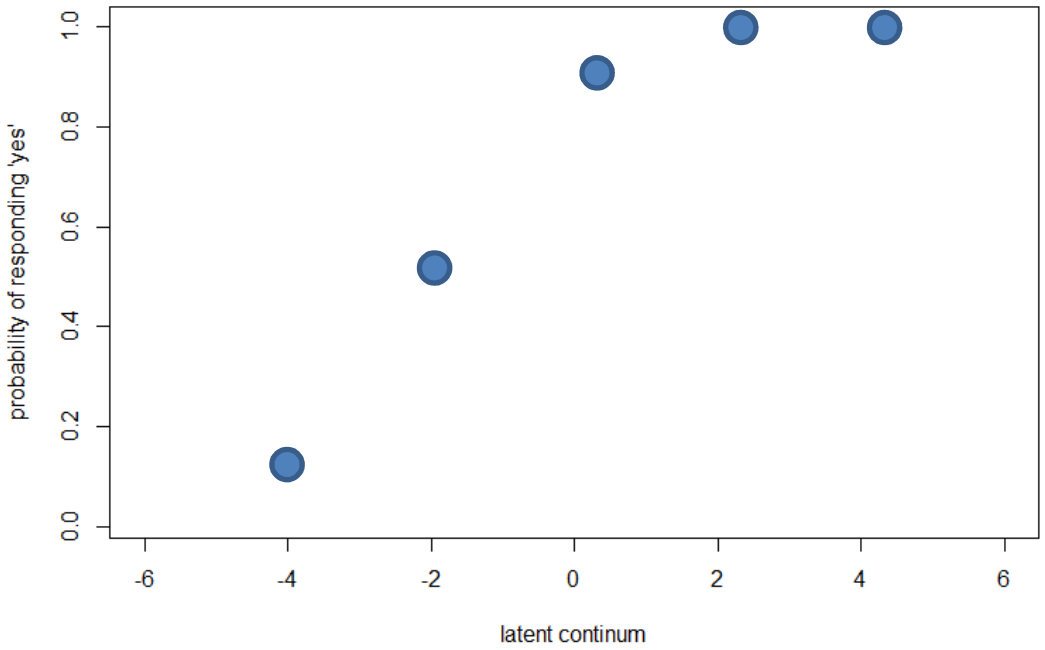

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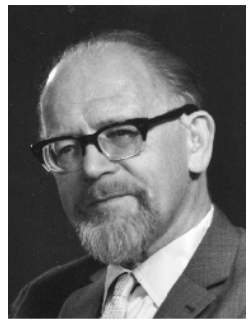
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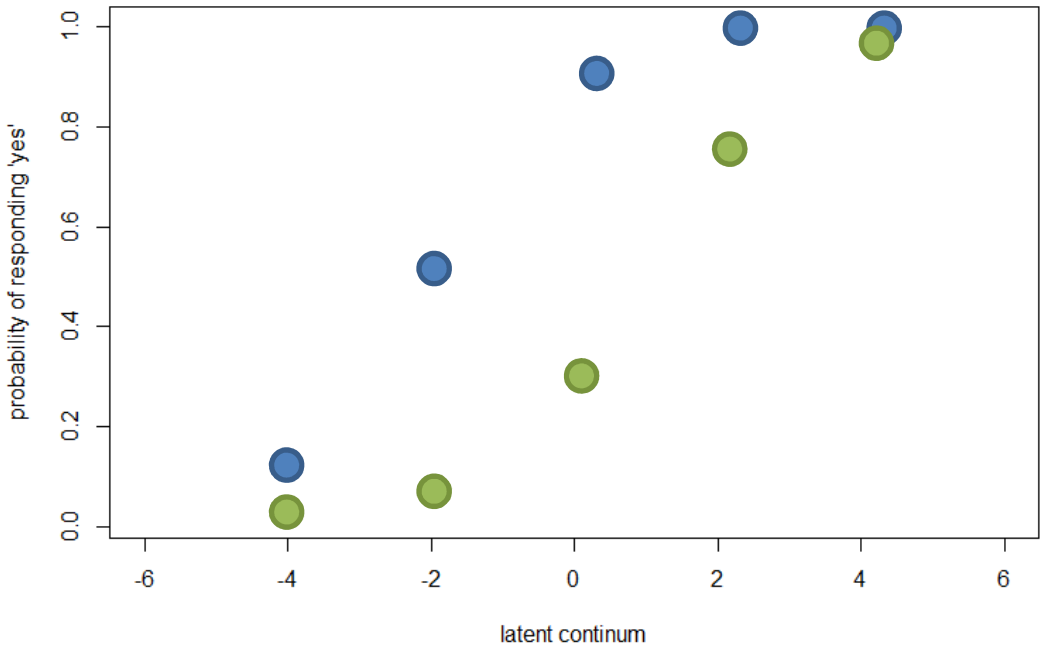

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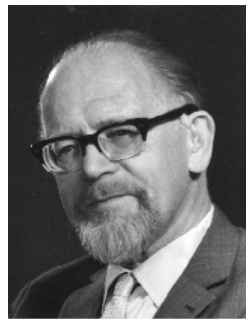
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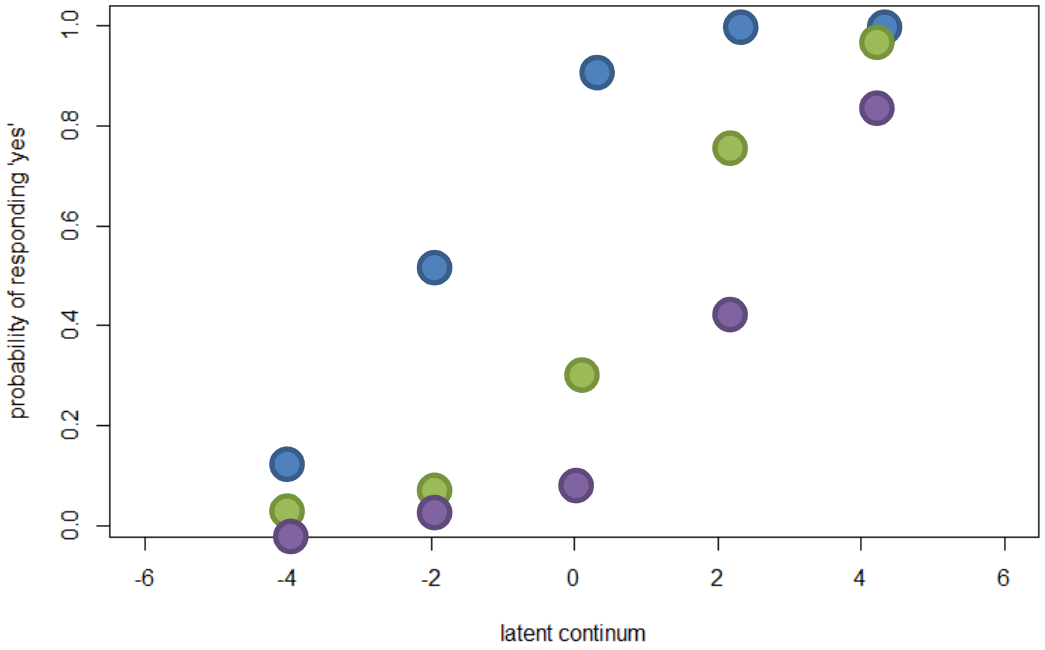

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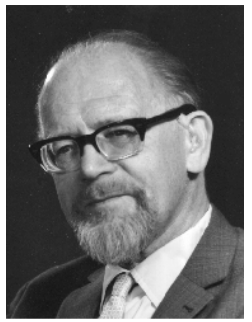
$$\Pr\{X_{ni} = 1\} = \frac{e^{\beta_n - \delta_i}}{1 + e^{\beta_n - \delta_i}}$$



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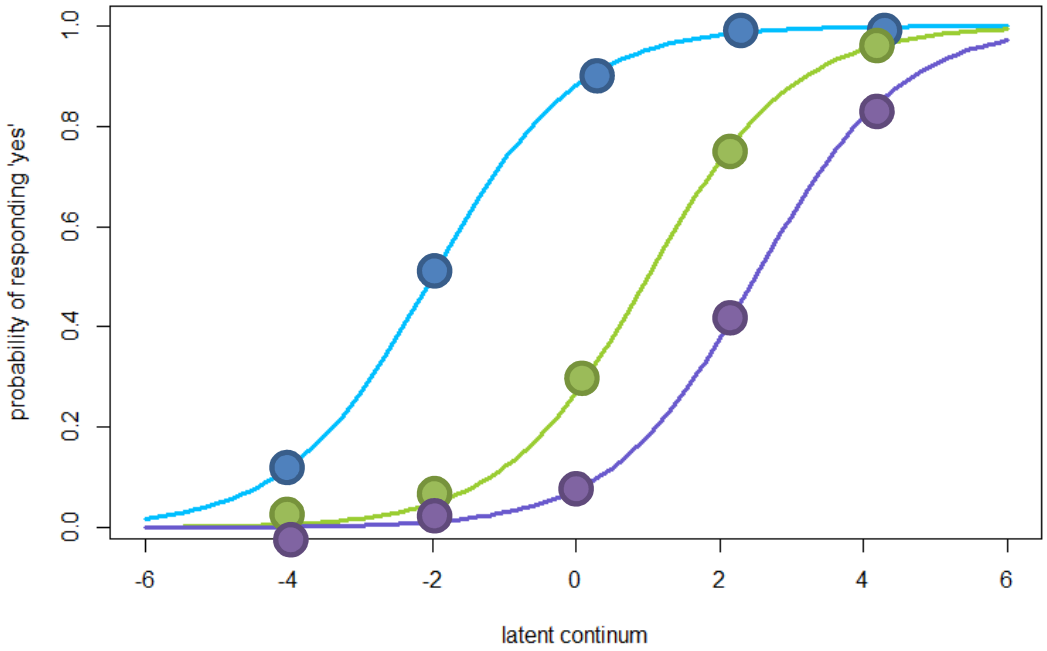

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A even less restrictive theory

- The probability of a "yes" answer depends only of the individuals' order on the latent trait

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The Mokken monotone homogeneity model (non parametric)

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1929-

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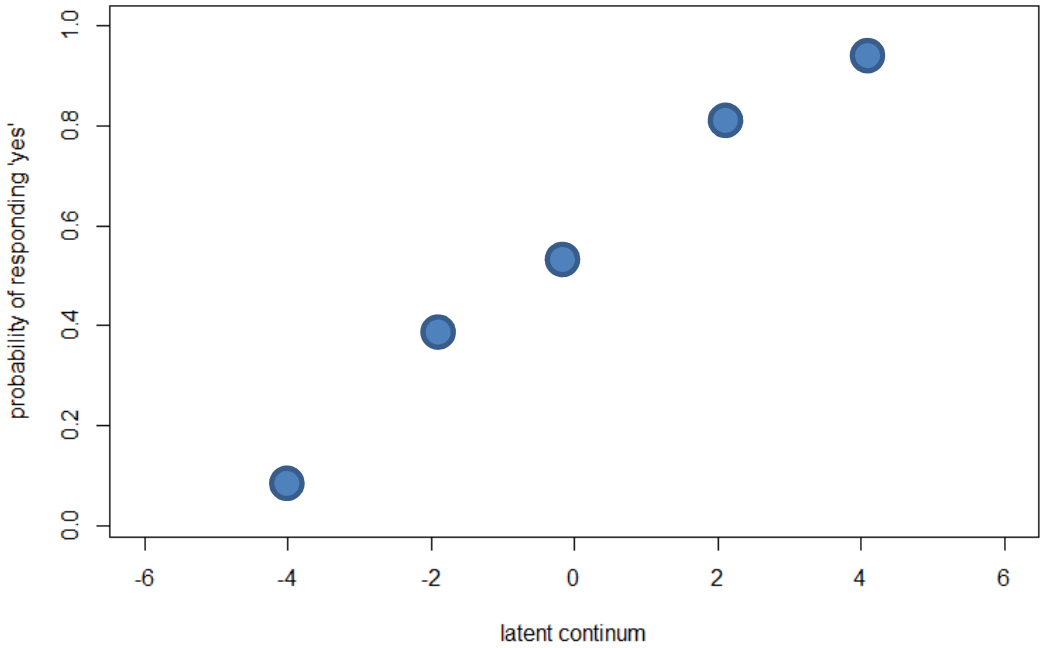

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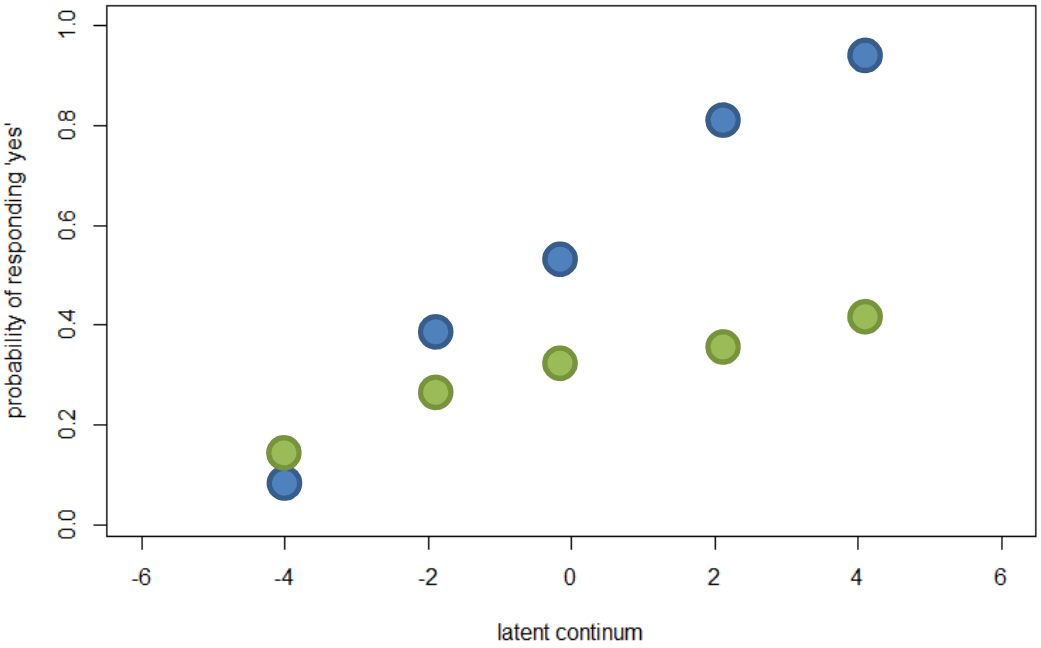

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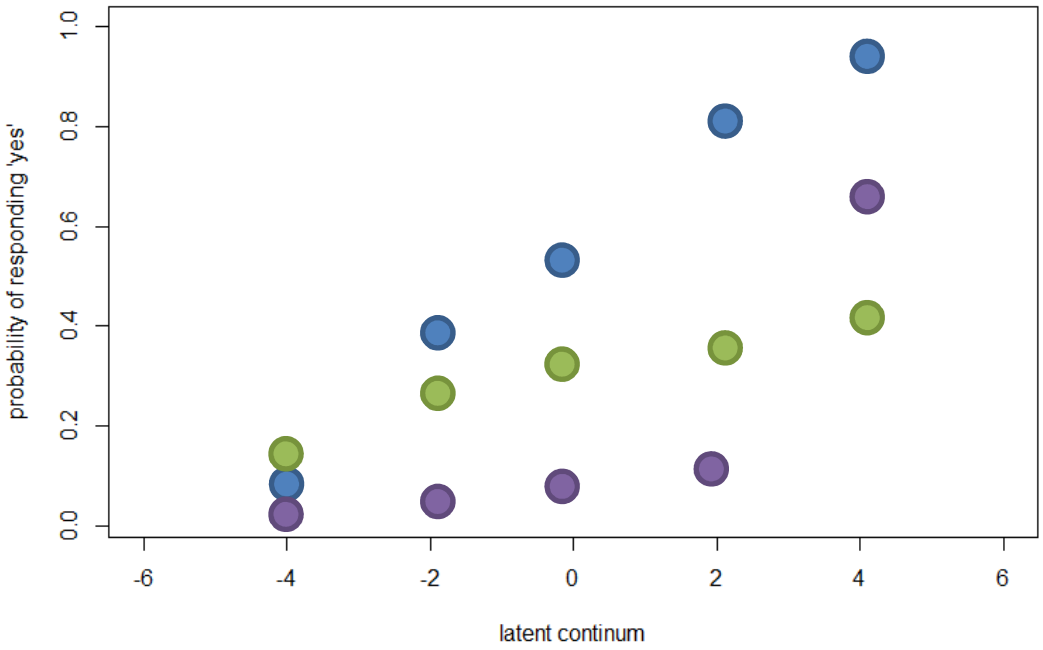

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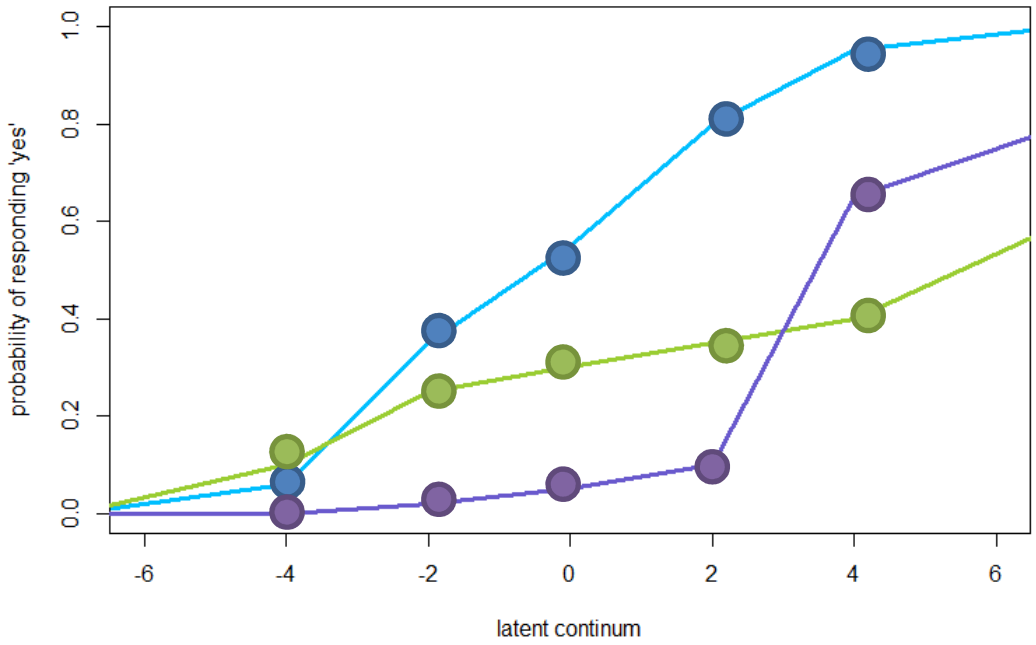

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A little more restrictive non parametric model

- The probability of a “yes” answer depends
  - of the individuals’ order on the latent trait
  - of the items’ difficulty order

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The Mokken double monotonicity model

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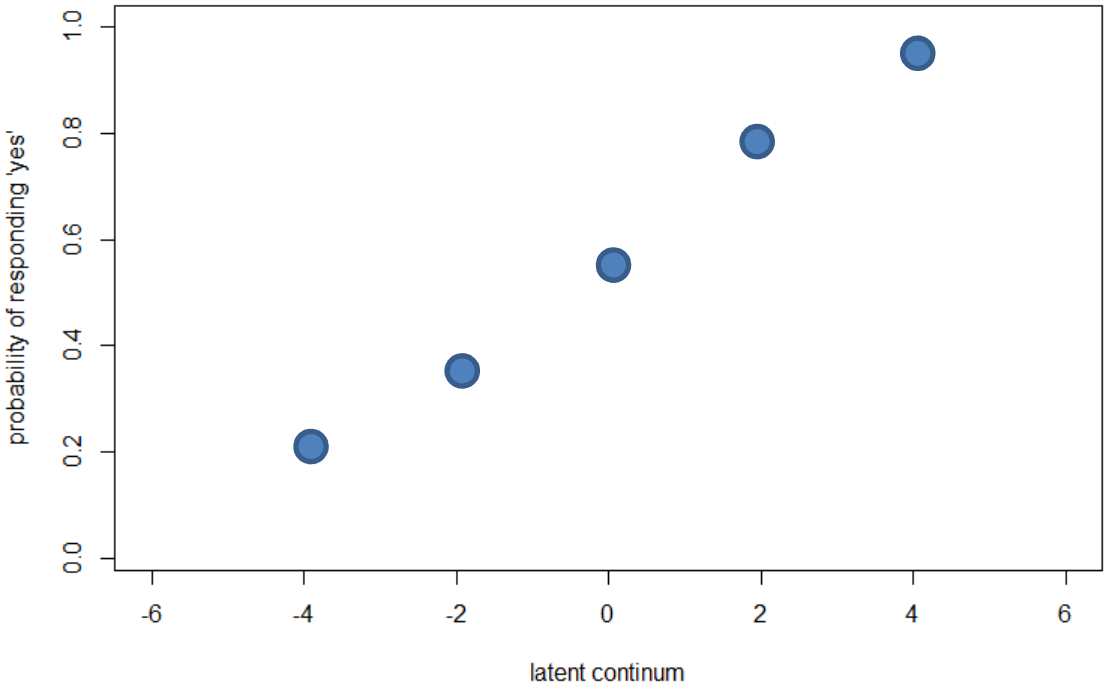

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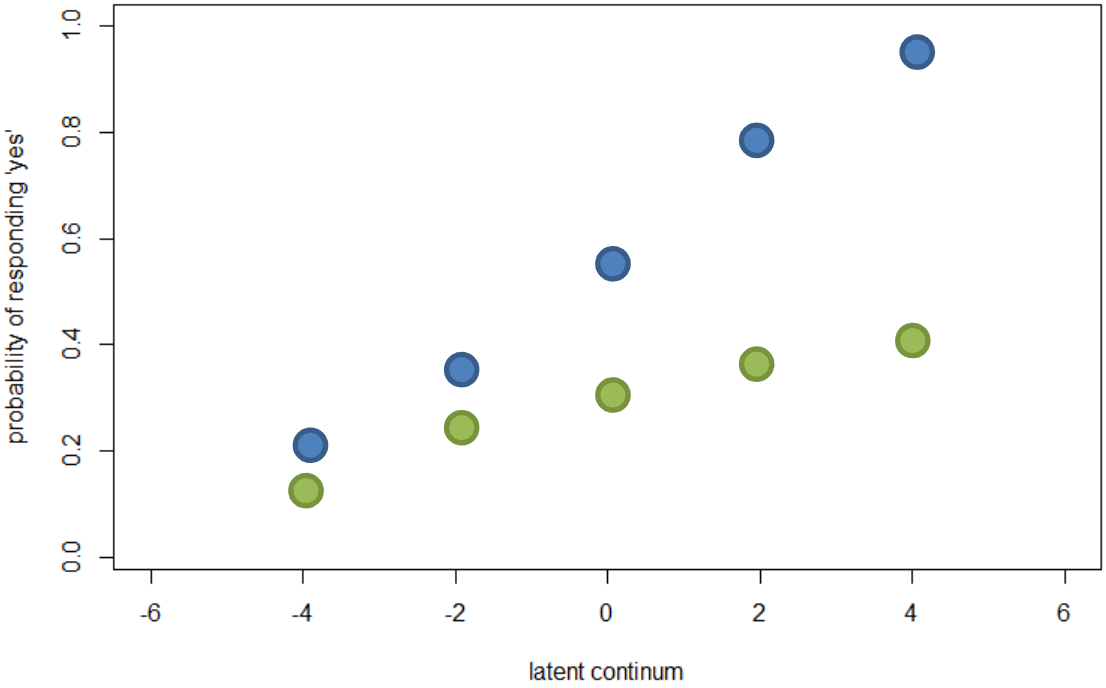

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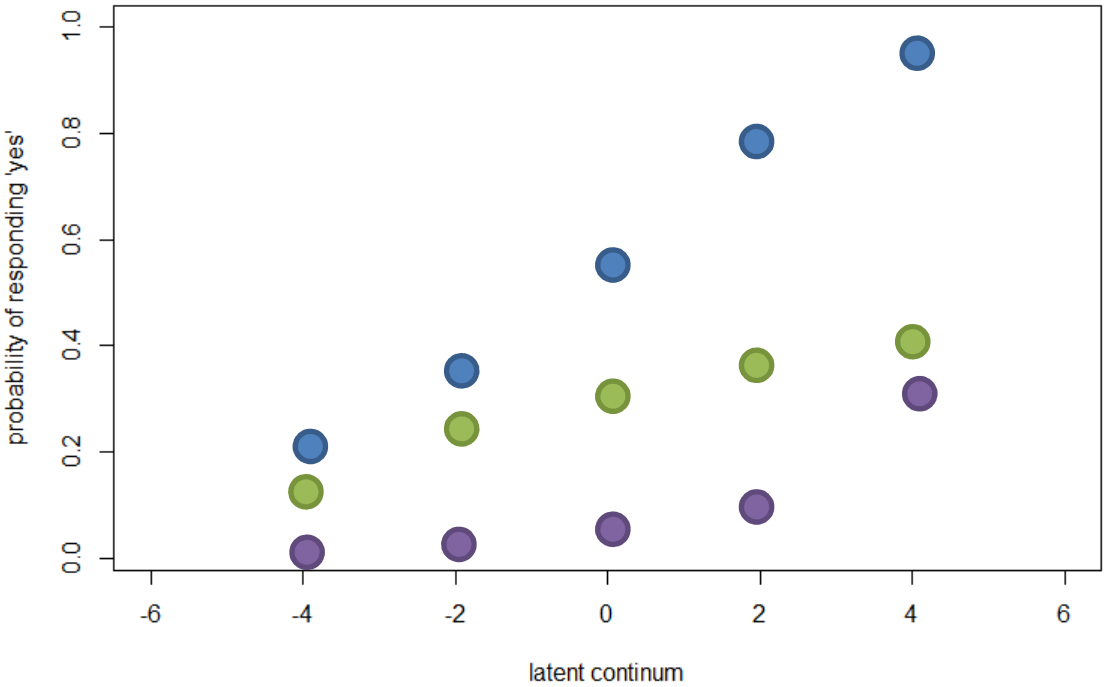

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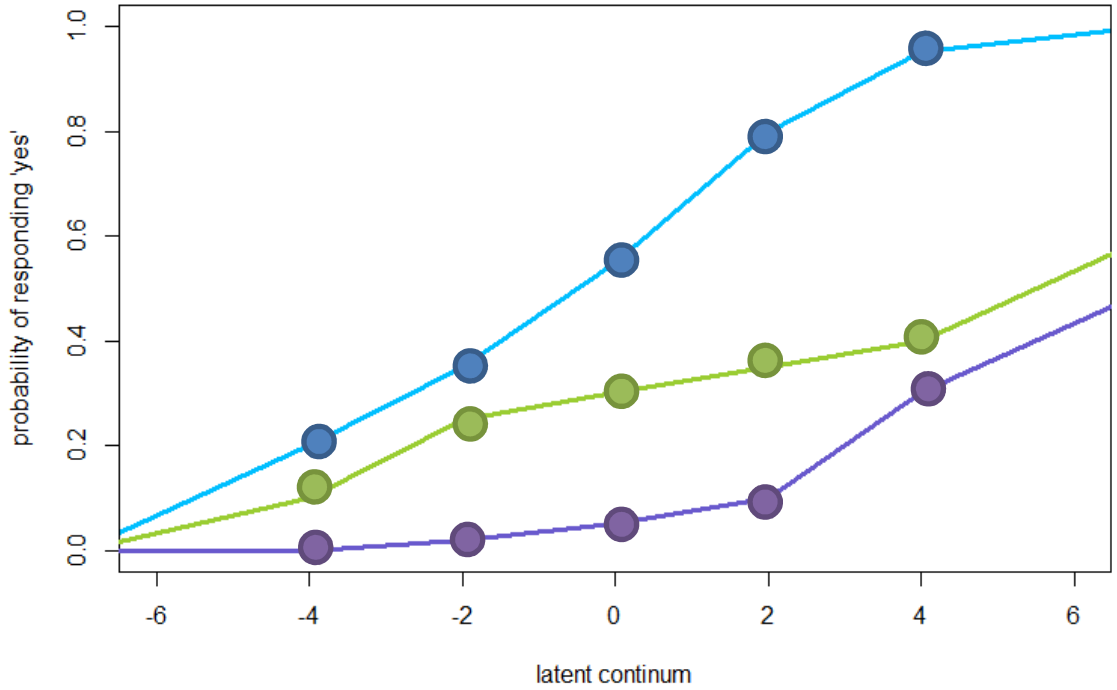

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How to check that the items form a Mokken scale?

An example : the “Physical and daily living needs” subscale of the Supportive Care Needs Survey (SCNS)

1. Pain
2. Lack of energy/tiredness
3. Feeling unwell a lot of the time
4. Work around the home
5. Not being able to do things you used to do

384 breast cancer patients from chemotherapy day hospital, the ambulatory radiotherapy service or in the surgery service (Paris, Institut Curie & Lausanne, Centre Hospitalier Universitaire Vaudois)

Boyes A., Girgis A. & Lecathelinais C. (2009) Brief assessment of adult cancer patients’ perceived needs: development and validation of the 34-item supportive Care Needs Survey (SCNS-SF34). *Journal of Evaluation in Clinical Practice, 15*, 602–606.
Brédart, A., Kop, J.-L., Griesser, A.-C., Zaman, K., Galmiche, M., Zimmers, S., Jacob, A., Berthet, V., Fiszer, C., & Dolbeault, S. (2012). Validation of the 34-item Supportive Care Needs Survey and 8-item Breast module French versions (SCNS-SF34-Fr and SCNS-BR8-Fr) in breast cancer patients. *European Journal of Cancer Care, 21*, 450-459.

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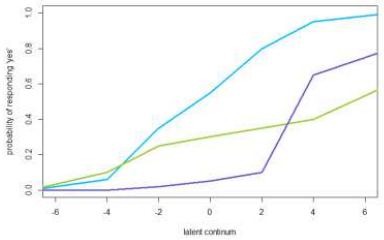
How to check that the items form a Mokken scale?

1) Are the items conform to the monotone homogeneity model?

Item 1 is conform to the monotone homogeneity model if the probability of responding « yes » to this item increases with the position on the latent trait



➔ An estimation of the position on the latent trait is needed !

➔ The responses to the other items of the scale will serve as estimates

5 items in the scale

- ➔ 4 items to estimate the position on the latent trait
- ➔ 5 positions (score = 0 for response « no » to all the 4 other items ; score = 1 for response « yes » to one of the other items and « no » to the 3 others, and so on)

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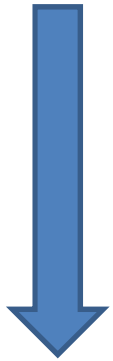
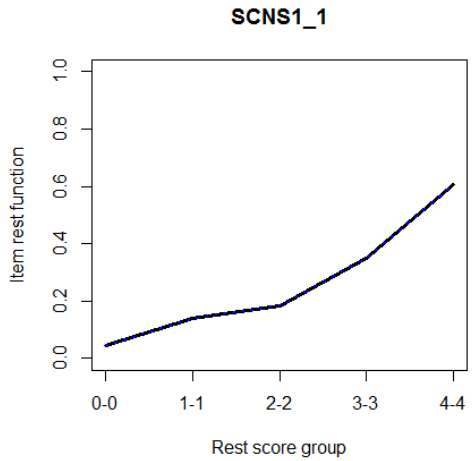
Estimation of latent trait (rest score)	% of reponses « yes » to item 1
0	4,4%
1	14,0%
2	18,5%
3	34,8%
4	60,6%





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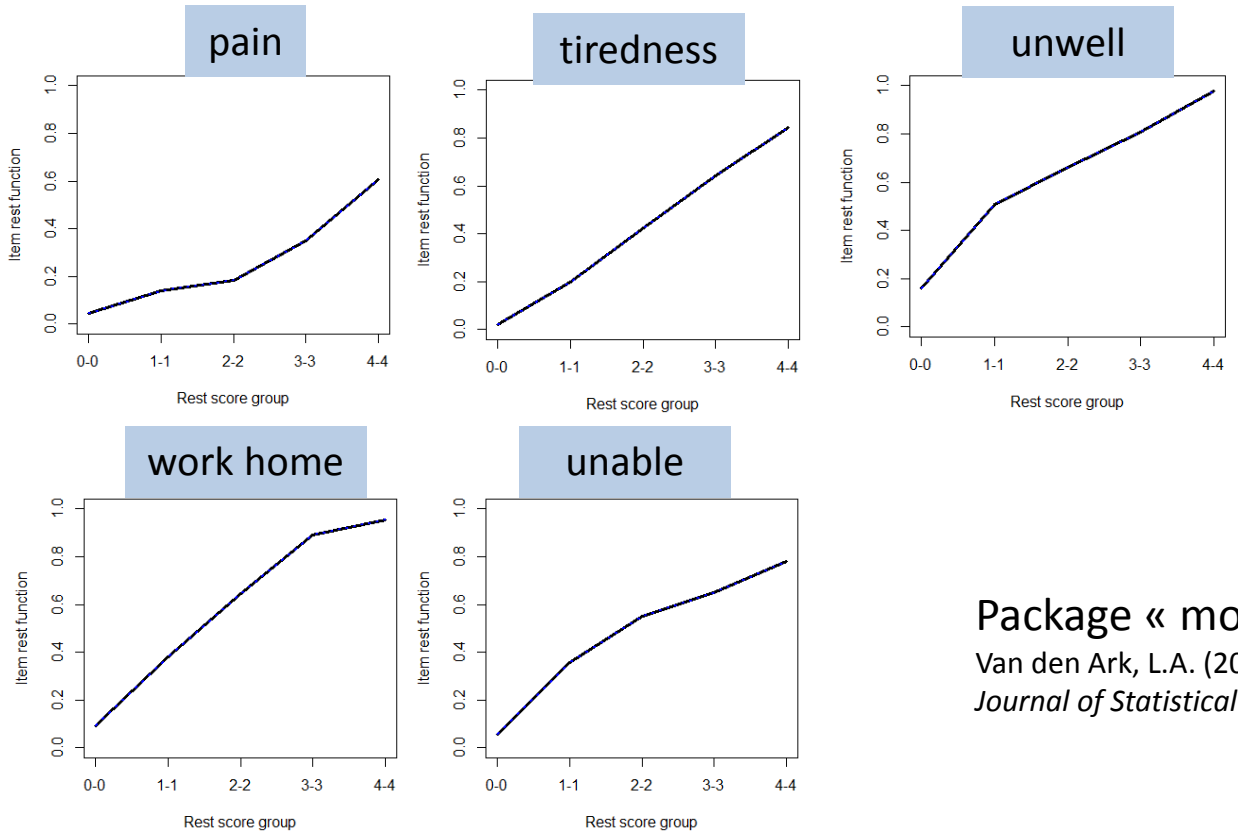
How to check that the items form a Mokken scale?

1) Are the items conform to the monotone homogeneity model ?



Package « mokken » of the R software

Van den Ark, L.A. (2007). Mokken scale analysis in R. *Journal of Statistical Software, 20* (11).

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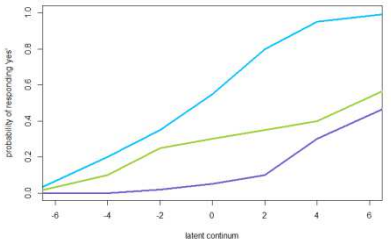
How to check that the items form a Mokken scale?

2) Are the items conform to the double monotonicity model?

Item 1 and item 2 are conform to the double monotonicity model if the probability of responding « yes » to item 2 is always greater to the probability of responding « yes » to item 1 (or the reverse)



➔ An estimation of the position on the latent trait is needed

➔ The responses to the other items of the scale will serve as estimates

5 items in the scale

➔ 3 items for the estimation of position on the latent trait

➔ 4 positions (score = 0 = for response « no » to all of the 3 other items ; score 1 for response « yes » to one of the other items and « no » to the 2 others, and so on)

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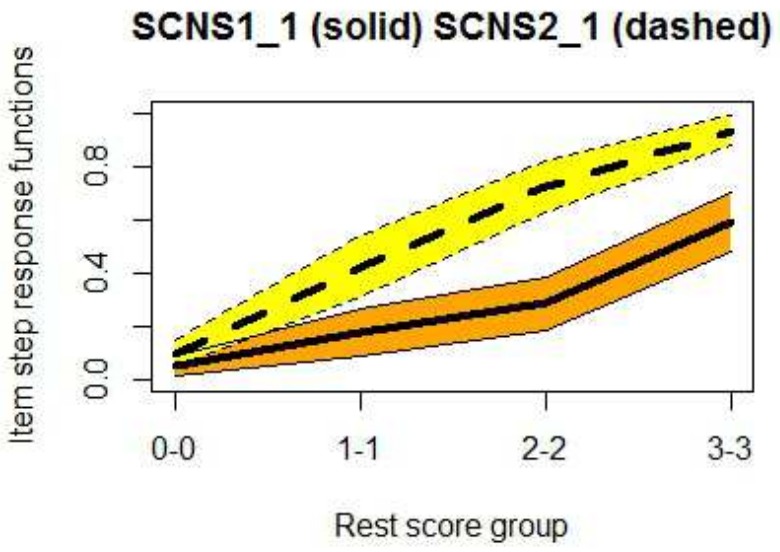
Estimation of latent trait (rest score)	% of reponses « yes » to item 1	% of reponses « yes » to item 2
0	5,3%	9,9%
1	17,8%	42,5%
2	28,6%	72,6%
3	59,2%	93,4%



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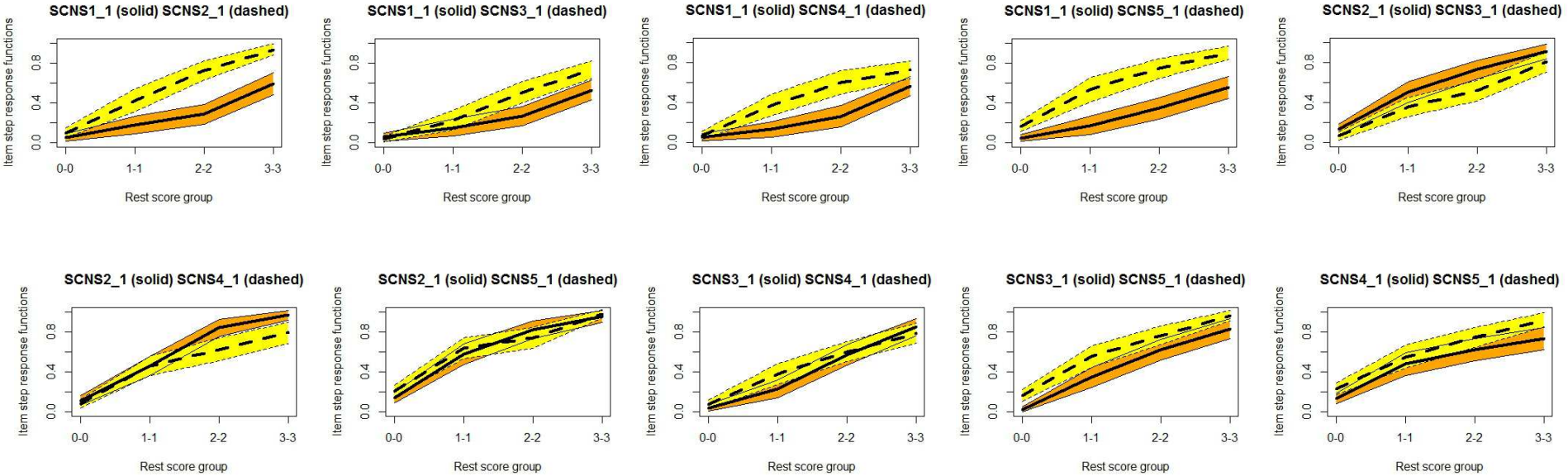

Package « mokken » of the R software

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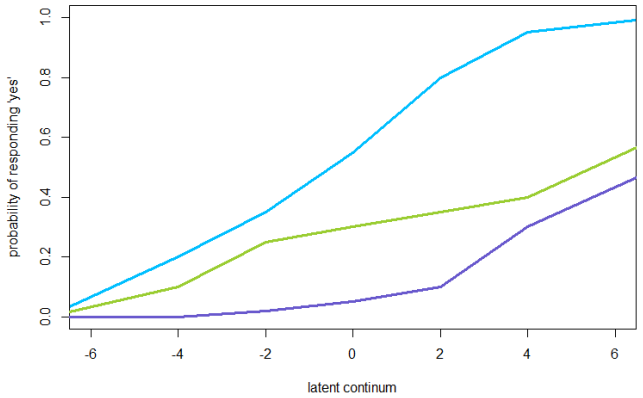
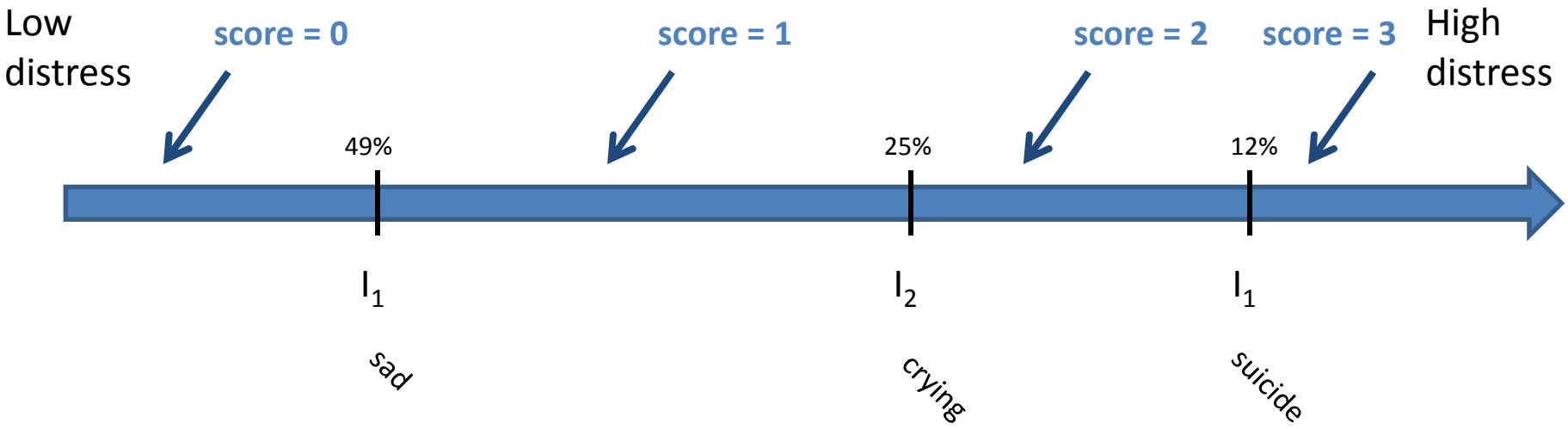
Double monotonicity and hierarchic scale

When the double monotonicity holds, there is an invariance item ordering (IIO) ➔ the scale is hierarchic



(|) (|) … (|)



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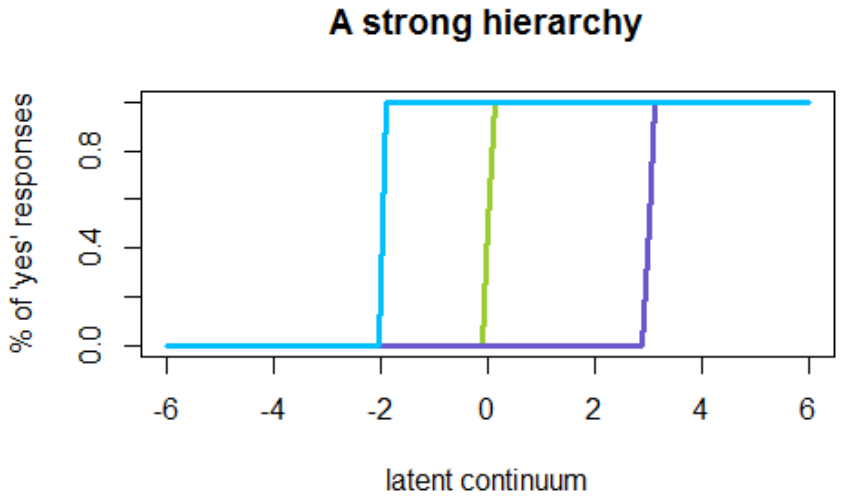
Various levels of hierarchy

A strict hierarchy

	sad	crying	suicide	%	%	%
CONFORM	Yes	Yes	Yes	10		
	Yes	Yes	No	25		
	Yes	No	No	45		
	No	No	No	20		
NOT CONFORM	Yes	No	Yes	0		
	No	Yes	Yes	0		
	No	Yes	No	0		
	No	No	Yes	0		



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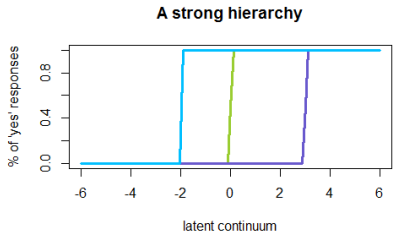
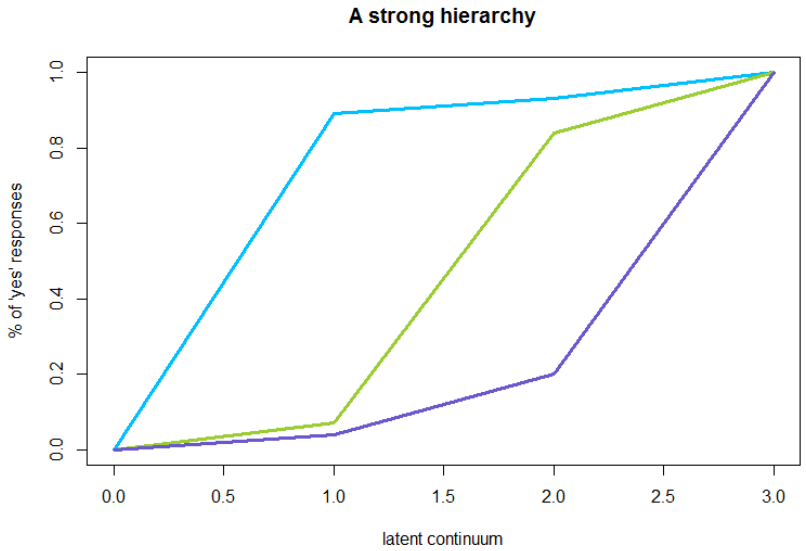
Various levels of hierarchy

A strong hierarchy

	sad	crying	suicide	%	%	%
CONFORM	Yes	Yes	Yes	10	10	
	Yes	Yes	No	25	20	
	Yes	No	No	45	40	
	No	No	No	20	20	
NOT CONFORM	Yes	No	Yes	0	4	
	No	Yes	Yes	0	1	
	No	Yes	No	0	3	
	No	No	Yes	0	2	





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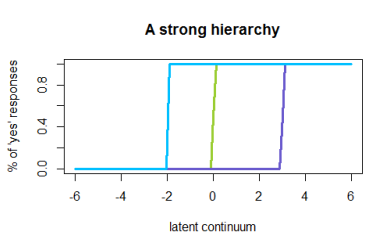
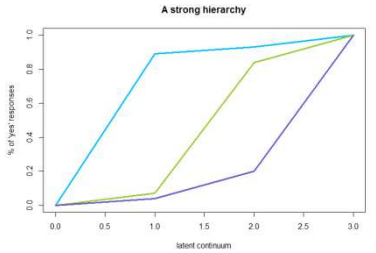
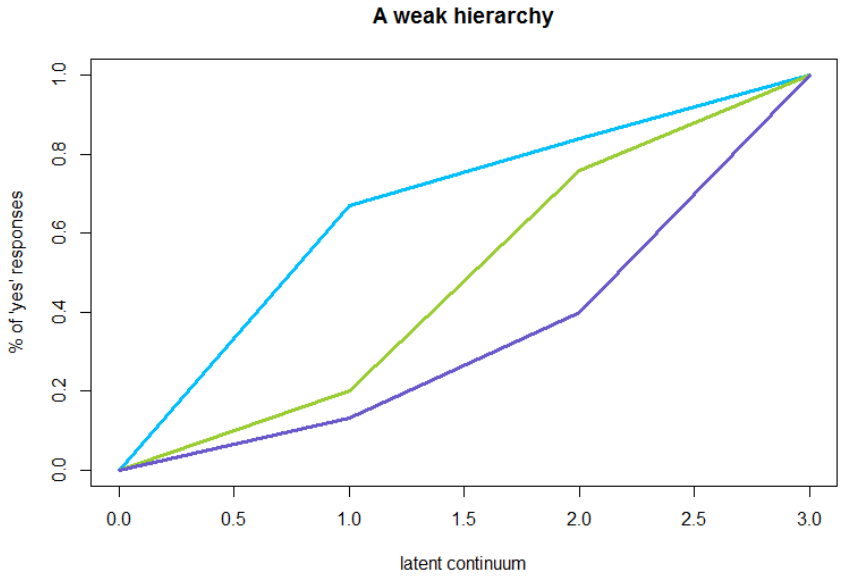
Various levels of hierarchy

A weak hierarchy

	sad	crying	suicide	%	%	%
CONFORM	Yes	Yes	Yes	10	10	10
	Yes	Yes	No	25	20	15
	Yes	No	No	45	40	30
	No	No	No	20	20	20
NOT CONFORM	Yes	No	Yes	0	4	6
	No	Yes	Yes	0	1	4
	No	Yes	No	0	3	9
	No	No	Yes	0	2	6







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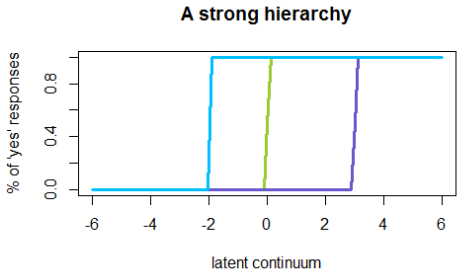
Various levels of hierarchy

	sad	crying	suicide	strict	strong	weak
CONFORM	Yes	Yes	Yes	10	10	10
	Yes	Yes	No	25	20	15
	Yes	No	No	45	40	30
	No	No	No	20	20	20
NOT CONFORM	Yes	No	Yes	0	4	6
	No	Yes	Yes	0	1	4
	No	Yes	No	0	3	9
	No	No	Yes	0	2	6

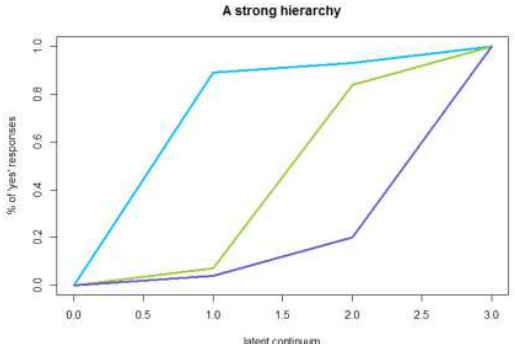
$H^T = 1{,}00$

0% of non conform patterns



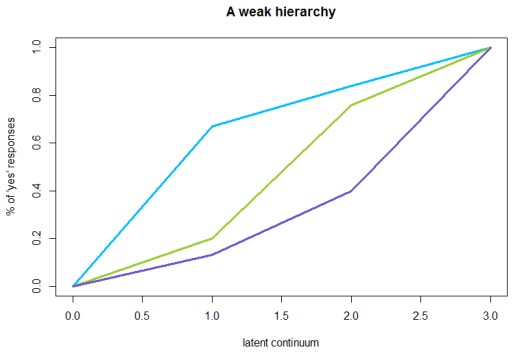
$H^T = 0{,}67$

10% of non conform patterns



$H^T = 0{,}24$

25% of non conform patterns



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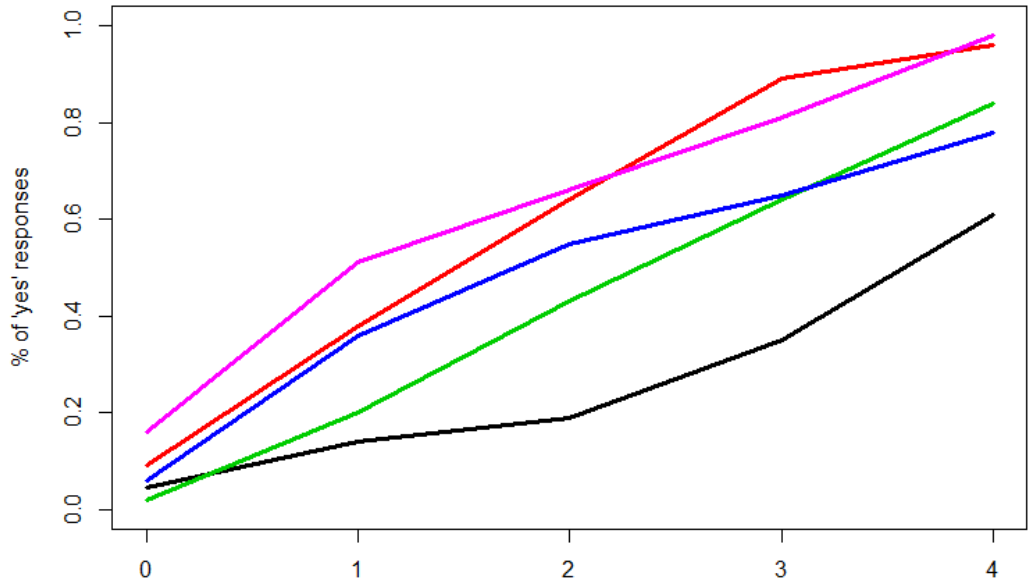
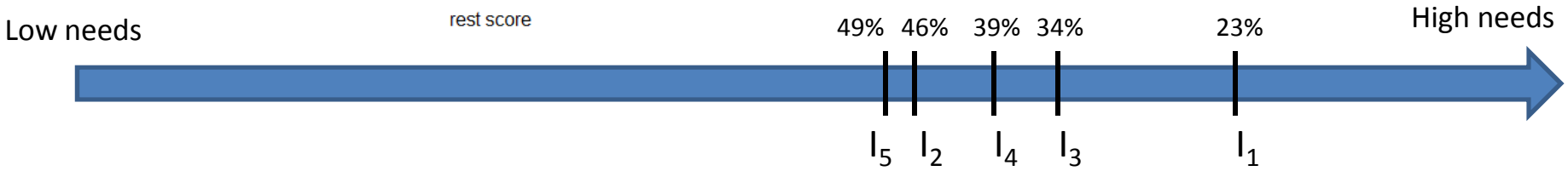
The "Physical and daily living needs subscale"

The hierarchy of the "Physical and daily living needs subscale" is a weak one

$H^T = 0,171$



1. Pain
2. Lack of energy/tiredness
3. Feeling unwell a lot of the time
4. Work around the home
5. Not being able to do things you used to do



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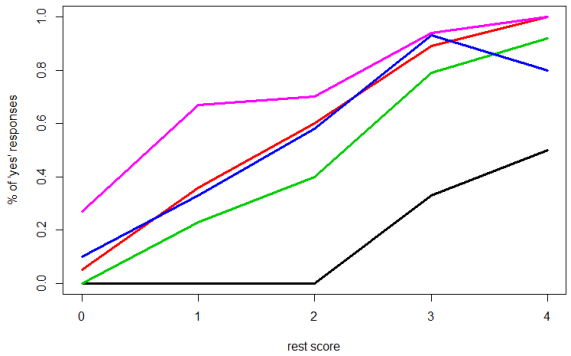
The “Physical and daily living needs subscale”

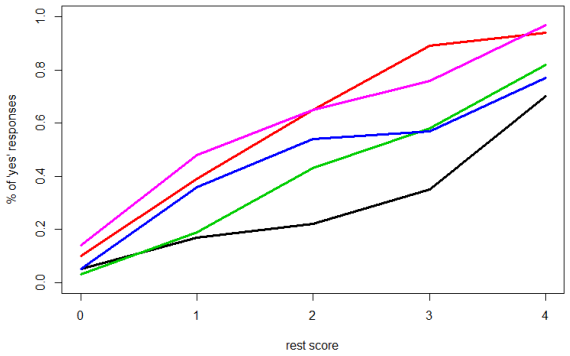
But the level of hierarchy is not the same for different categories of patients

Metastatic cancer



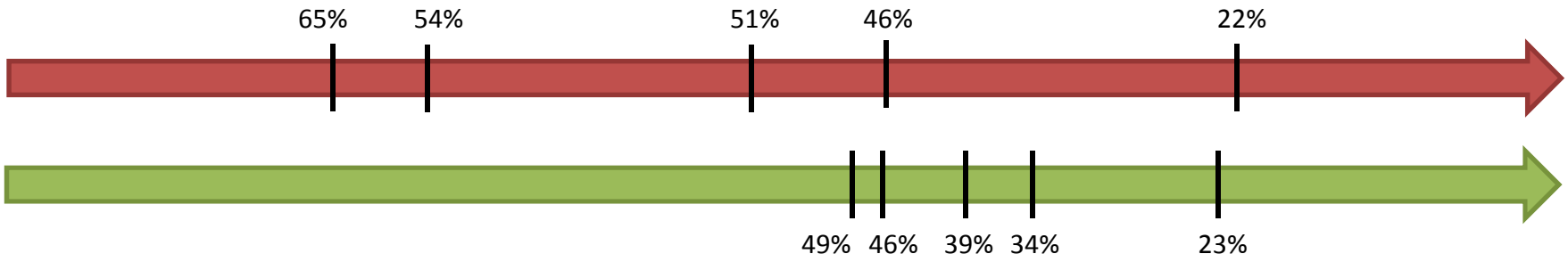
1. Pain
2. Lack of energy/tiredness
3. Feeling unwell a lot of the time
4. Work around the home
5. Not being able to do things you used to do

Localized cancer



H^T = 0,505
24% of non conform patterns

H^T = 0,171
35% of non conform patterns



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Conclusion

- Mokken scaling is an easy way to approach Item Response Models
- The Mokken scale is less demanding than the Rasch model
- The double monotonicity model has an interesting property: the (probalistic) hierarchy of the scale
- Hierarchical scales are probably more useful for clinicians
  - interpretation of scores
  - identification of patients with non conform pattern

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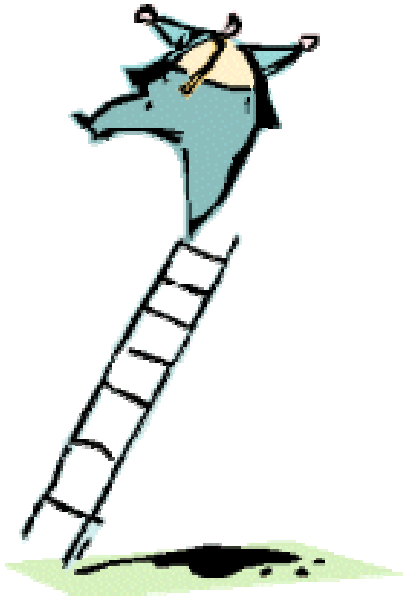

There is a problem with the hierarchical scale !

THANK YOU for YOUR ATTENTION